Math 250 Trig Review Practice – Day 1

1) Evaluate for $f(x) = \cos 2x$

   a. $f\left(\frac{\pi}{6}\right)$

   b. $f\left(\frac{\pi}{2}\right)$

   c. $f\left(\frac{5\pi}{8}\right)$

2) Solve for $x \in [0, 2\pi)$:

   a. $\sec x = \sqrt{2}$

   b. $\cot x = -1$

   c. $\csc x = -\frac{2\sqrt{3}}{3}$

3) Solve $2\sin x \cos x = \sqrt{3}\sin x$ for

   a. $x \in [0, 2\pi)$

   b. all x

4) Solve. $\sin(2x) = -\frac{\sqrt{3}}{2}$ for $x \in [0, 2\pi)$

5) Determine if $g(x) = x\cot x$ is odd, even, or neither.

Math 250 Trig Review Objectives

Objectives

1) Angles & radian measure
   a. If using graphing calculator, be sure to check radian versus degree mode

2) Evaluate trig functions
   a. Using triangles with a given trig ratio or given triangle
   b. For any multiple of $\frac{\pi}{4}$ or $\frac{\pi}{6}$, positive or negative, using standard reference triangles
      i. 30-60-90 triangle; half of equilateral triangle, short side is opposite smallest angle.
      ii. 45-45-90 triangle; isosceles right triangle has two equal sides.
   c. Using unit circle for quadrantal angles and signs of trig values

3) Solve trig equations
   a. If necessary, re-write using identities to get one trig function type
   b. If degree 1, isolate trig function
   c. If degree 2 or higher, set = 0, factor completely, set factors equal to zero, isolate trig function.
   d. Find values of argument (angle).
   e. If variable is not isolated in the argument, isolate variable.
   f. If period of graph < standard period, repeatedly isolate variable using larger coterminal angles
   g. Check solutions are in given domain: $[0,2\pi)$ or if $(-\infty,\infty)$, use k or n and **Z**

4) Graphs of trig functions
   a. Period
   b. Transformations: Amplitude, vertical shift, horizontal shift (phase change)
   c. Equations of asymptotes

5) Domains and ranges of trig functions:
   a. $\sin(x)$ has domain $(-\infty,\infty)$ and range $[-1,1]$.
   b. $\cos(x)$ has domain $(-\infty,\infty)$ and range $[-1,1]$.
   c. $\tan(x)$ has domain $\left\{x : x \neq \frac{\pi}{2} + k\pi, k \in Z\right\}$ and range $(-\infty,\infty)$
   d. $\cot(x)$ has domain $\{x : x \neq k\pi, k \in Z\}$ and range $(-\infty,\infty)$
   e. $\sec(x)$ has domain $\left\{x : x \neq \frac{\pi}{2} + k\pi, k \in Z\right\}$ and range $(-\infty,-1] \cup [1,\infty)$
   f. $\csc(x)$ has domain $\{x : x \neq k\pi, k \in Z\}$ and range $(-\infty,-1] \cup [1,\infty)$

6) Recognize and use trig identities to simplify or evaluate at sums or differences of known angles, especially
   a. Reciprocal and Quotient Identities
   b. Pythagorean Identities
   c. Double-Angle and Half-Angle Identities
   d. Odd/Even (or Sign) Identities
   e. Law of Sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos C$

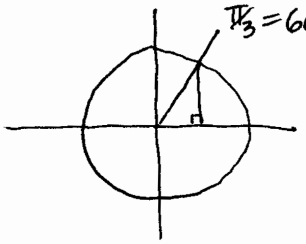
M250 Trig Review – 1

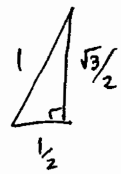
① Evaluate $f(x) = \cos(2x)$

a) $f\left(\frac{\pi}{6}\right)$ replace x by $\frac{\pi}{6}$

$= \cos\left(2 \cdot \frac{\pi}{6}\right)$ ← order of operations inside out

$= \cos\left(\frac{\pi}{3}\right)$ $2 \cdot \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$ (reduce $\frac{2}{6}$)



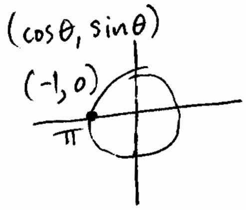


$30° - 60° - 90°$ Δ

$= \boxed{\frac{1}{2}}$

b) $f\left(\frac{\pi}{2}\right)$

$= \cos\left(2 \cdot \frac{\pi}{2}\right)$



quadrantal angle

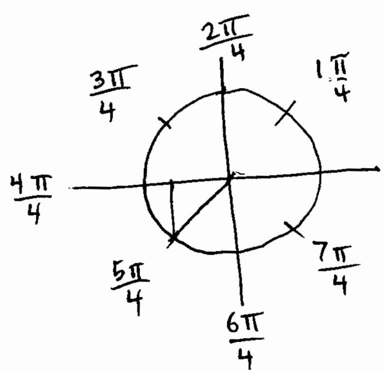
$= \cos \pi$

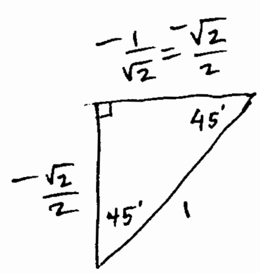
$= \boxed{-1}$

c) $f\left(\frac{5\pi}{8}\right)$

$= \cos\left(2 \cdot \frac{5\pi}{8}\right)$ $2 \cdot \frac{5}{8} = \frac{5}{4}$

$= \cos\left(\frac{5}{4}\pi\right)$





$= \boxed{\frac{-\sqrt{2}}{2}}$ or $\boxed{-\frac{1}{\sqrt{2}}}$

M250 Trig Review – 1

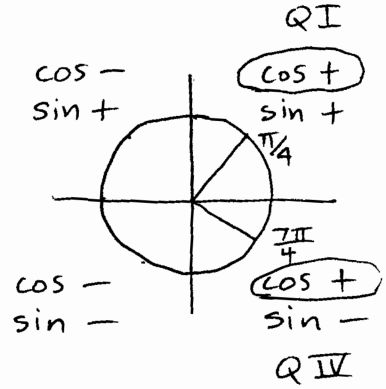
(2) Solve for $x \in [0, 2\pi)$ one revolution on the unit circle

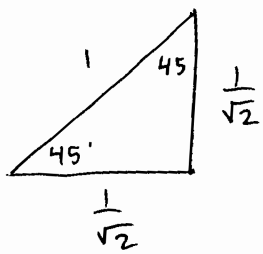
a) $\sec x = \sqrt{2}$

$$\frac{1}{\cos x} = \sqrt{2}$$

$$1 = \sqrt{2} \cdot \cos x$$

$$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = \cos x$$





$$\boxed{x = \frac{\pi}{4}, \frac{7\pi}{4}}$$

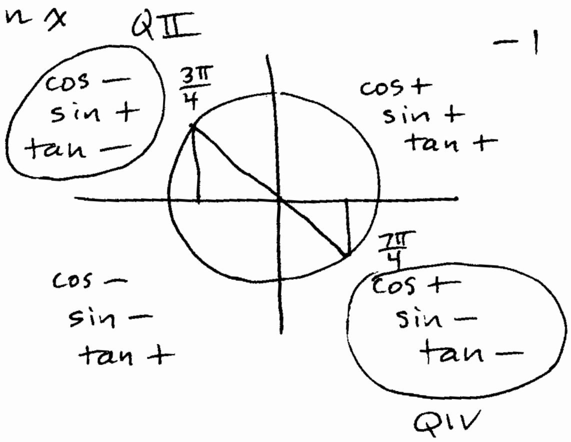
b) $\cot x = -1$ OR $\frac{1}{\tan x} = -1$

$$\frac{\cos x}{\sin x} = -1$$

$$1 = -\tan x$$

$$\cos x = -\sin x$$

$$-1 = \tan x$$



$$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}}$$

M250 Trig Review – 1

② c.

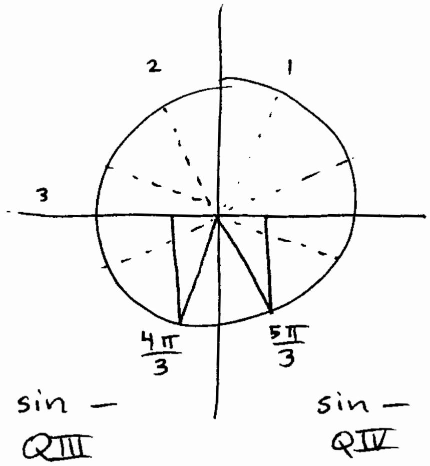
$$\csc x = \frac{-2\sqrt{3}}{3} = \frac{-2}{\sqrt{3}}$$

$$\frac{1}{\sin x} = \frac{-2}{\sqrt{3}}$$

$$\sqrt{3} = -2 \sin x$$

$$-\frac{\sqrt{3}}{2} = \sin x$$

$$\boxed{x = \frac{4\pi}{3}, \frac{5\pi}{3}}$$



y-coordinate = sine
$\Rightarrow$ longer side $\frac{\sqrt{3}}{2}$
$\approx .866$

③ Solve $2 \sin x \cos x = \sqrt{3} \sin x$

a) $x \in [0, 2\pi)$
b) all x $\leftarrow$ meaning $(-\infty, \infty)$ or all $\mathbb{R}$eal numbers x.

$$2 \sin x \cos x = \sqrt{3} \sin x$$

CAUTION: Do NOT DIVIDE BY $\sin x$ or solutions may be lost ☹

set = 0:

$$2 \sin x \cos x - \sqrt{3} \sin x = 0.$$

factor:

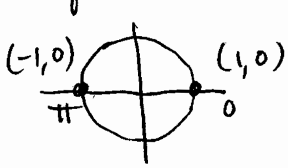
$$\sin x \left(2 \cos x - \sqrt{3}\right) = 0$$

set factors = 0:

$$\sin x = 0 \qquad 2\cos x - \sqrt{3} = 0$$

Solve resulting equations:

$\sin x = 0$



$x = 0, \pi$

SOLVE A SIMPLER PROBLEM:

Hint: Same strategy as for solving
$2x^3 = 3x^2$

set = 0: $2x^3 - 3x^2 = 0$
factor: $x^2(2x-3) = 0$
set factors = 0:
$x^2 = 0 \qquad 2x - 3 = 0$
solve resulting eqn's
$x = 0 \qquad x = \frac{3}{2}$

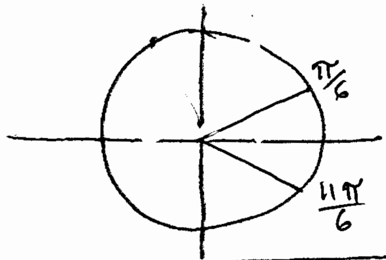
CAUTION: Do NOT DIVIDE BOTH SIDES by x^2 or the $x=0$ solution would be lost. ☹

M250 Trig Review - 1

$$2\cos x - \sqrt{3} = 0$$ isolate trig function

$$2\cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

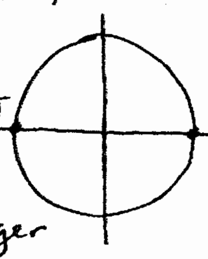


$x = \pi, \pi$

Solutions: $\boxed{x = 0, \pi, \frac{\pi}{6}, \frac{11\pi}{6}}$ ← sometimes MML/MXL cares what <u>order</u> you write these answers. Watch for blue instructions.

ex: $\boxed{x = 0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}}$
if must be in order

b) all x



... $7\pi, 5\pi, 3\pi, \pi$ | $0, 2\pi, 4\pi, 6\pi$ ...

$\pi + 2\pi n$, n an integer | $0 + 2\pi n$, n an integer

Each of these solutions is repeated by its coterminal angles.
add 2π, add 4π, add 6π, etc.

But these two angles are equally spaced around the unit circle, so we can combine the lists:

$$0, \pi, 2\pi, 3\pi, 4\pi, \ldots$$

<u>$x = n\pi$ where n is an integer</u>

$x = \frac{\pi}{6}$ and $\frac{11\pi}{6}$ are not equally spaced around the unit circle, so each one must be written separately

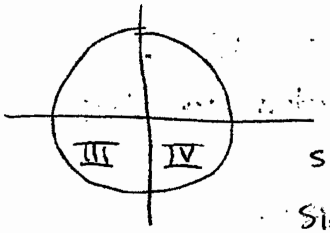
$$\frac{\pi}{6} + 2\pi n \quad \text{and} \quad \frac{11\pi}{6} + 2\pi n$$

solutions $\boxed{x = n\pi, \frac{\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n \text{ where } n \text{ is an integer}}$

M250 Trig Review - 1

4 Solve.

$$\sin 2x = -\frac{\sqrt{3}}{2} \text{ for } x \in [0, 2\pi)$$

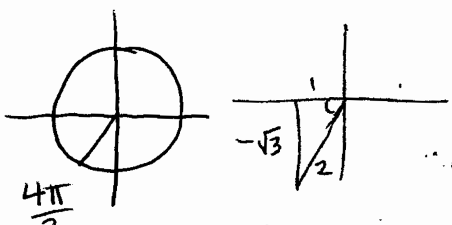


sine is negative in quadrants III & IV.

$\sin\theta = -\sqrt{3}/2$

$\theta = \frac{4\pi}{3}, \frac{10\pi}{3}$

$2x = \frac{4\pi}{3}$

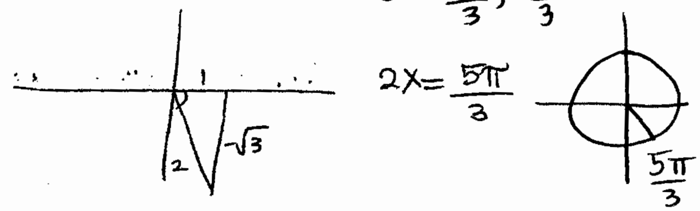


reference angle $\frac{\pi}{3}$

$\sin\theta = -\sqrt{3}/2$

$\theta = \frac{5\pi}{3}, \frac{11\pi}{3}$

$2x = \frac{5\pi}{3}$



reference angle $\frac{\pi}{3}$

$2x = \frac{4\pi}{3}$

$\boxed{x = \frac{2\pi}{3}}$

$2x = \frac{5\pi}{3}$

$\boxed{x = \frac{5\pi}{6}}$

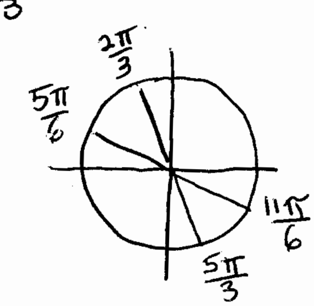
sin(2x) has period π -- it oscillates more frequently (2x) than sin(x), so it has more solutions. — use coterminal angles

$2x = \frac{4\pi}{3} + 2\pi$

$2x = \frac{4\pi}{3} + \frac{6\pi}{3}$

$2x = \frac{10\pi}{3}$

$\boxed{x = \frac{5\pi}{3}}$



$2x = \frac{5\pi}{3} + 2\pi$

$2x = \frac{5\pi}{3} + \frac{6\pi}{3}$

$2x = \frac{11\pi}{3}$

$\boxed{x = \frac{11\pi}{6}}$

Solutions: $\boxed{\frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}}$

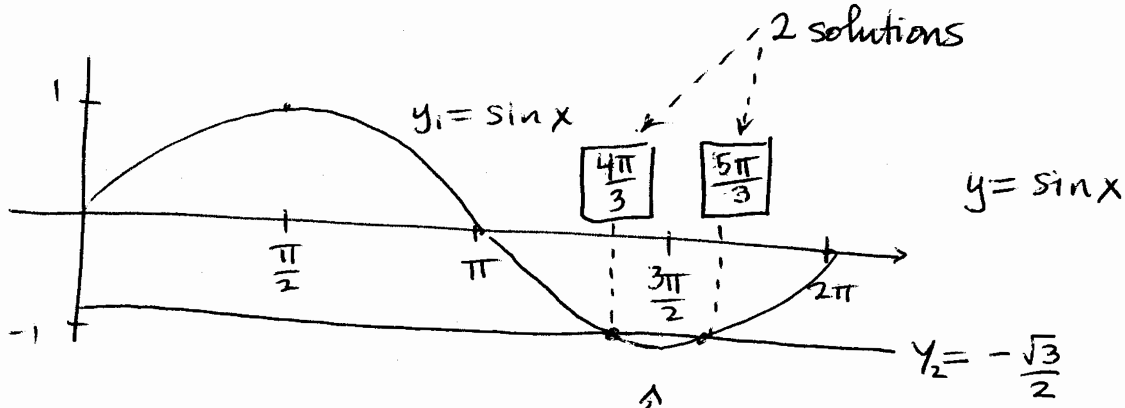
check with GC! (ZOOM TRIG)

(4)

Notice the difference between the graphs & intersections

$y_1 = \sin x$

$y_2 = -\sqrt{3}/2$



$-\frac{\sqrt{3}}{2} \approx -.9$

2 intersections at reference angles

Versus:

$y_1 = \sin(2x)$

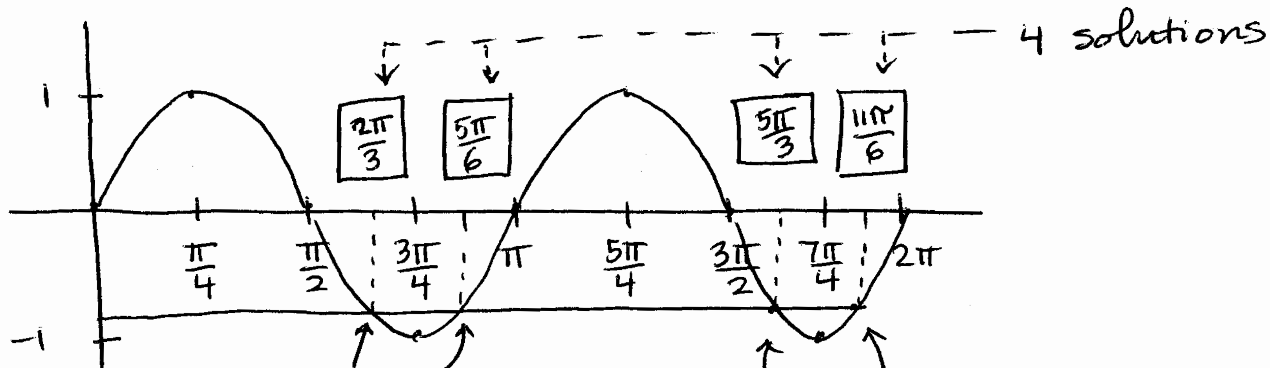
$y_2 = -\sqrt{3}/2$

period $2x = 2\pi$

$x = \pi$

set argument = original period

solve for x to get new period



two intersections compressed closer to the origin

two additional intersections now appear in the second cycle.

period π between $x = \frac{2\pi}{3}$ and $x = \frac{5\pi}{3}$

ditto: period π between $x = \frac{5\pi}{6}$ and $x = \frac{11\pi}{6}$.

(5) Determine if $g(x) = x \cot x$ is odd, even, or neither.

odd: $f(-x) = -f(x)$
even: $f(-x) = f(x)$
neither: doesn't work out to either $f(x)$ or $-f(x)$.

$g(-x) = -x \cdot \cot(-x)$ — subst $-x$ for x

$= -x \dfrac{\cos(-x)}{\sin(-x)}$ — identity for $\cot(x) = \dfrac{\cos x}{\sin x}$

$= -x \cdot \dfrac{\cos x}{-\sin x}$ — identity $\cos(-x) = \cos x$, $\sin(-x) = -\sin x$

$= x \cdot \dfrac{\cos x}{\sin x}$ — simplify signs

$= x \cdot \cot x$ — $\cot x$ identity

$= g(x)$

$g(-x) = g(x)$ $\boxed{\text{even}}$

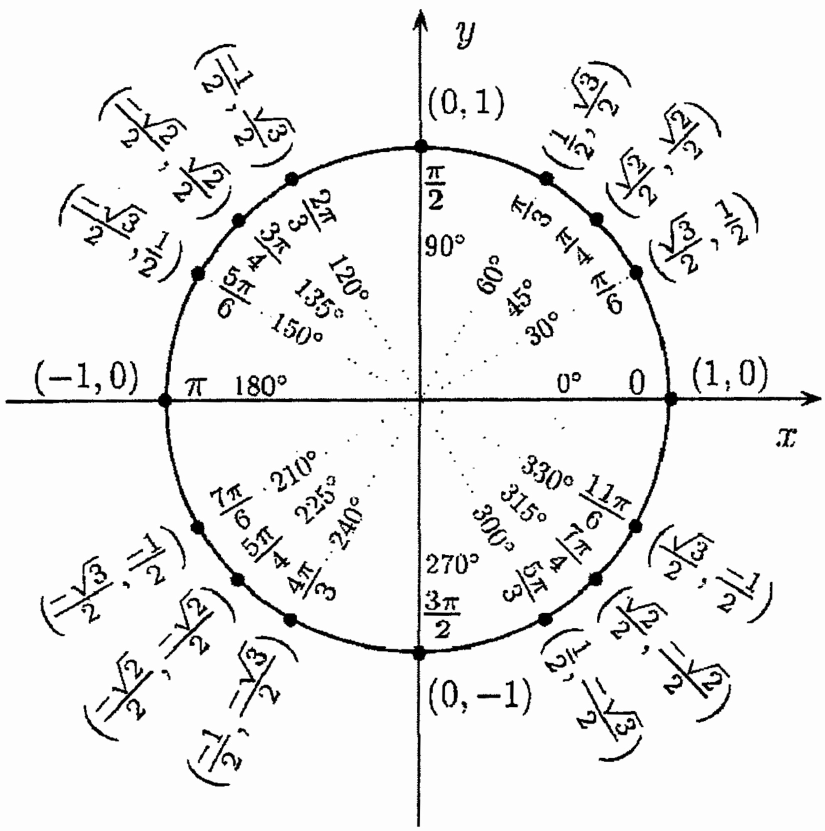
Trigonometry Summary

Right Triangle Trigonometry

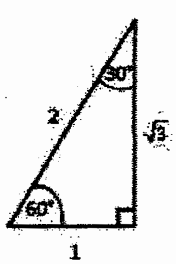
$$\sin\theta = \frac{\textit{opposite}\quad (y)}{\textit{hypotenuse}\quad (r)} \qquad \cos\theta = \frac{\textit{adjacent}\quad (x)}{\textit{hypotenuse}\quad (r)} \qquad \tan\theta = \frac{\textit{opposite}\quad (y)}{\textit{adjacent}\quad (x)}$$

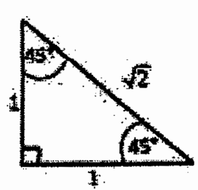
$$\csc\theta = \frac{\textit{hypotenuse}\quad (r)}{\textit{opposite}\quad (y)} \qquad \sec\theta = \frac{\textit{hypotenuse}\quad (r)}{\textit{adjacent}\quad (x)} \qquad \cot\theta = \frac{\textit{adjacent}\quad (x)}{\textit{opposite}\quad (y)}$$

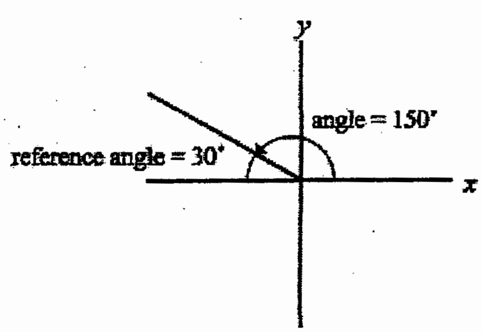
Unit Circle: Radius 1, Coordinates (x, y) on the unit circle are $(\cos\theta, \sin\theta)$.

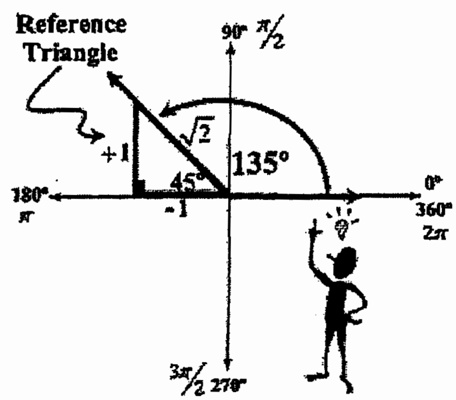


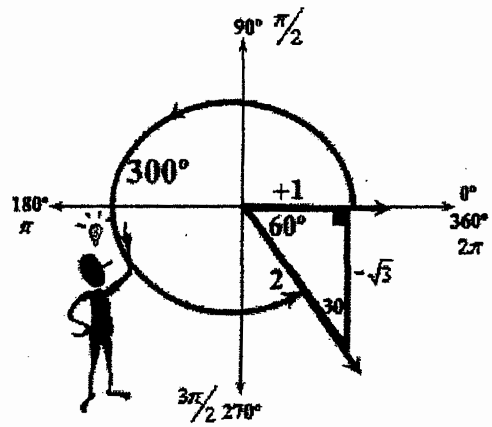
Radian Measure: One radian is the angle that intercepts an arc one unit long in a circle whose radius is 1.
π radians $= 180°$, so we can use the proportion $\frac{D}{R} = \frac{180}{\pi}$, where D is degree and R is radians, to convert.











Reciprocal Identities

$\sin(\theta)=\frac{1}{\csc(\theta)}$ $\cos(\theta)=\frac{1}{\sec(\theta)}$ $\tan(\theta)=\frac{1}{\cot(\theta)}$

$\csc(\theta)=\frac{1}{\sin(\theta)}$ $\sec(\theta)=\frac{1}{\cos(\theta)}$ $\cot(\theta)=\frac{1}{\tan(\theta)}$

Quotient Identities are also called the Tangent and Cotangent Identities

$\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)}$ $\cot(\theta)=\frac{\cos(\theta)}{\sin(\theta)}$

Sign Identities are also called **Odd and Even Function Properties**
Cosine and Secant are even, and all others are odd.

$\sin(-\theta)=-\sin(\theta)$ $\cos(-\theta)=\cos(\theta)$ $\tan(-\theta)=-\tan(\theta)$

$\csc(-\theta)=-\csc(\theta)$ $\sec(-\theta)=\sec(\theta)$ $\cot(-\theta)=-\cot(\theta)$

Cofunction Identities

$\sin\left(\frac{\pi}{2}-x\right)=\cos(x)$ $\tan\left(\frac{\pi}{2}-x\right)=\cot(x)$ $\sec\left(\frac{\pi}{2}-x\right)=\csc(x)$

$\cos\left(\frac{\pi}{2}-x\right)=\sin(x)$ $\cot\left(\frac{\pi}{2}-x\right)=\tan(x)$ $\csc\left(\frac{\pi}{2}-x\right)=\sec(x)$

$\sin(90°-x)=\cos(x)$ $\tan(90°-x)=\cot(x)$ $\sec(90°-x)=\csc(x)$

$\cos(90°-x)=\sin(x)$ $\cot(90°-x)=\tan(x)$ $\csc(90°-x)=\sec(x)$

Pythagorean Identities

$\sin^2\theta+\cos^2\theta=1$ $1+\tan^2\theta=\sec^2\theta$ $1+\cot^2\theta=\csc^2\theta$

Arc length $s=r\theta$ **Sector area** $A=\frac{1}{2}r^2\theta$

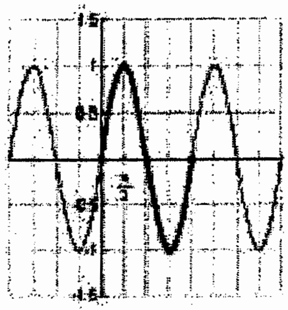
Graphs of the Six Trigonometric Functions

$y = \sin x$

Domain:
All Reals

Range:
$[-1, 1]$

Period: 2π

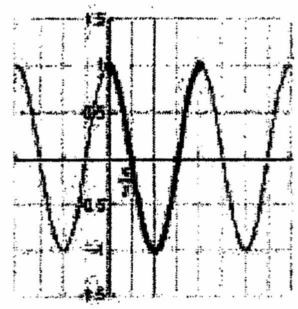


$y = \cos x$

Domain:
All Reals

Range:
$[-1, 1]$

Period: 2π

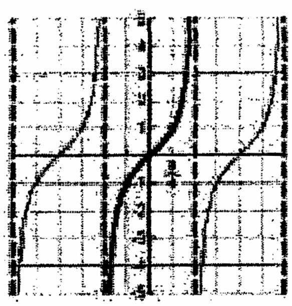


$y = \tan x$

Domain:
All $x \neq \frac{\pi}{2} + n\pi$

Range:
All Reals

Period: π

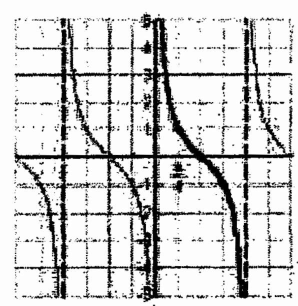


$y = \cot x$

Domain:
All $x \neq n\pi$

Range:
All Reals

Period: π

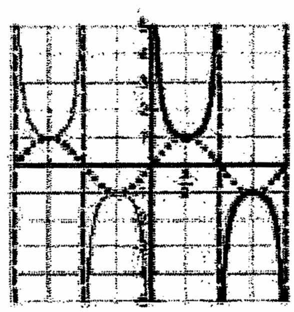


$y = \csc x$

Domain:
All $x \neq n\pi$

Range:
$(-\infty, -1] \cup [1, \infty)$

Period: 2π

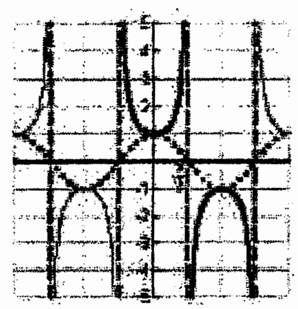


$y = \sec x$

Domain:
All $x \neq \frac{\pi}{2} + n\pi$

Range:
$(-\infty, -1] \cup [1, \infty)$

Period: 2π



Law of Sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$, *a, b, c* are side lengths, and *A, B, C* are angles opposite those sides.

Law of Cosines:

$a^2 = b^2 + c^2 - 2bc\cos A$ $\qquad$ $b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Note: the last version strongly resembles the Pythagorean theorem for the sides of a right triangle.

Sum and Difference Formulas

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\qquad$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Period, Amplitude, Vertical and Horizontal Shifts, Vertical Asymptotes

	Period	Amplitude	Vertical shift	Horizontal shift	Vertical Asymptotes
$y=\sin x$	2π	1	None	None	None
$y=a\sin(bx+c)+d$	$\frac{2\pi}{b}$	a	d	$-\frac{c}{b}$	None
$y=\cos x$	2π	1	None	None	None
$y=a\cos(bx+c)+d$	$\frac{2\pi}{b}$	a	d	$-\frac{c}{b}$	None
$y=\tan x$	π	None	None	None	$\frac{\pi}{2}+k\pi, k\in Z$
$y=a\tan(bx+c)+d$	$\frac{\pi}{b}$	None	d	$-\frac{c}{b}$	$\frac{1}{b}\left(\frac{\pi}{2}-c+k\pi\right), k\in Z$
$y=\cot x$	π	None	None	None	$k\pi, k\in Z$
$y=a\cot(bx+c)+d$	$\frac{\pi}{b}$	None	d	$-\frac{c}{b}$	$\frac{1}{b}(-c+k\pi), k\in Z$
$y=\sec x$	2π	None	None	None	$\frac{\pi}{2}+k\pi, k\in Z$
$y=a\sec(bx+c)+d$	$\frac{2\pi}{b}$	None	d	$-\frac{c}{b}$	$\frac{1}{b}\left(\frac{\pi}{2}-c+k\pi\right), k\in Z$
$y=\csc x$	2π	None	None	None	$k\pi, k\in Z$
$y=a\csc(bx+c)+d$	$\frac{2\pi}{b}$	None	d	$-\frac{c}{b}$	$\frac{1}{b}(-c+k\pi), k\in Z$

Double-Angle Formulas can be derived from Sum formulas by substituting another α for β

Derive additional versions for cosine by substituting the Pythagorean Identity.

$\sin(2\alpha)=2\sin\alpha\cos\alpha$ $\qquad$ $\tan(2\alpha)=\dfrac{2\tan\alpha}{1-\tan^2\alpha}$

$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha$ $\qquad$ $\cos(2\alpha)=2\cos^2\alpha-1$ $\qquad$ $\cos(2\alpha)=1-2\sin^2\alpha$

Half-Angle Formulas are derived from the Power Reducing Formulas by taking square roots, then replace $2u$ by θ and u by $\frac{\theta}{2}$. You MUST know the quadrant of $\frac{\theta}{2}$ to determine + or − when there's a $\pm$.

$\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\dfrac{1-\cos(\theta)}{2}}$ $\qquad$ $\cos\left(\frac{\theta}{2}\right)=\pm\sqrt{\dfrac{1+\cos(\theta)}{2}}$

$\tan\left(\frac{\theta}{2}\right)=\pm\sqrt{\dfrac{1-\cos(\theta)}{1+\cos(\theta)}}$ $\qquad$ $\tan\left(\frac{\theta}{2}\right)=\dfrac{\sin\theta}{1+\cos\theta}$ $\qquad$ $\tan\left(\frac{\theta}{2}\right)=\dfrac{1-\cos\theta}{\sin\theta}$

Product-to-Sum Formulas are derived by adding or subtracting two sum or difference formulas

$\sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$ $\qquad$ $\cos\alpha\sin\beta=\frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$ continued

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)] \qquad \sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

Sum-to-Product Identities are derived by solving the linear system $\begin{cases} x=\alpha+\beta \\ y=\alpha-\beta \end{cases}$ for α and β, substituting the resulting expressions for α and β in the Product-to-Sum Formulas, and multiplying by 2.

$$\sin x+\sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \qquad \sin x-\sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x+\cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} \qquad \cos x-\cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

Triangle Area Formulas

If a triangle has side lengths *a*, *b*, and *c*, with opposite angles A, B, and C, then the area of the triangle is

$$Area = \frac{1}{2}bc\sin A \qquad Area = \frac{1}{2}ac\sin B \qquad Area = \frac{1}{2}ab\sin C$$

If a triangle has side lengths *a*, *b*, and *c*, and $s=\frac{1}{2}(a+b+c)$, then $Area=\sqrt{s(s-a)(s-b)(s-c)}$ (Heron's)

Power Reducing Formulas are derived from the Double-Angle formulas for cosine

For example: $\cos(2u)=2\cos^2 u-1=1-2\sin^2 u$ Subtract 1 from both sides

$\cos(2u)-1=-2\sin^2 u$, then divide both sides by -2 and rearrange to get $\frac{1-\cos(2u)}{2}=\sin^2 u$

$$\sin^2 u=\frac{1-\cos(2u)}{2} \qquad \cos^2 u=\frac{1+\cos(2u)}{2} \qquad \tan^2 u=\frac{1-\cos(2u)}{1+\cos(2u)}$$

Domain and Range of Inverse Trig Functions

$$y=\sin^{-1}x \quad \begin{cases} -1\le x\le 1 \\ -\frac{\pi}{2}\le y\le\frac{\pi}{2} \end{cases} \qquad y=\tan^{-1}x \quad \begin{cases} -\infty\le x\le\infty \\ -\frac{\pi}{2}\le y\le\frac{\pi}{2} \end{cases} \qquad y=\csc^{-1}x \quad \begin{cases} x\le -1 \ or\ x\ge 1 \\ -\frac{\pi}{2}\le y\le\frac{\pi}{2},\ y\ne 0 \end{cases}$$

$$y=\cos^{-1}x \quad \begin{cases} -1\le x\le 1 \\ 0\le y\le\pi \end{cases} \qquad y=\cot^{-1}x \quad \begin{cases} -\infty\le x\le\infty \\ 0\le y\le\pi \end{cases} \qquad y=\sec^{-1}x \quad \begin{cases} x\le -1 \ or\ x\ge 1 \\ 0\le y\le\pi,\ y\ne\frac{\pi}{2} \end{cases}$$

Greek alphabet

A	α	*alpha*	H	η	*eta*	N	ν	*nu*	T	τ	*tau*
B	β	*beta*	Θ	θ	*theta*	Ξ	ξ	*xi*	Y	υ	*upsilon*
Γ	γ	*gamma*	I	ι	*iota*	O	o	*omicron*	Φ	ϕ	*phi*
Δ	δ	*delta*	K	κ	*kappa*	Π	π	*pi*	X	χ	*chi*
E	ε	*epsilon*	Λ	λ	*lambda*	P	ρ	*rho*	Ψ	ψ	*psi*
Z	ς	*zeta*	M	μ	*mu*	Σ	σ	*sigma*	Ω	ω	*omega*